Critical Point Flyby

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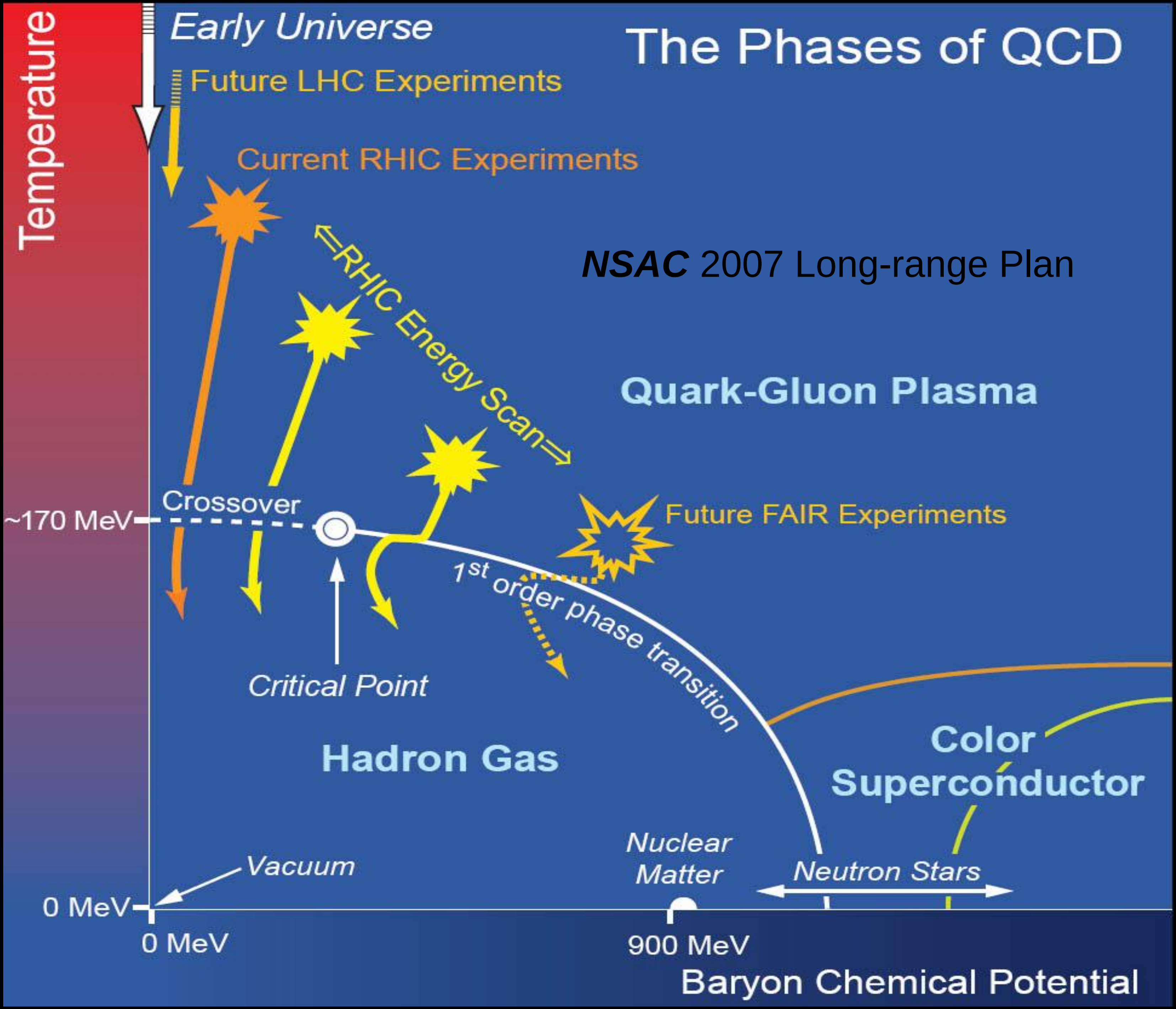
BertschFest, INT, September 2012

Phase Structure of QCD: Diverse Studies Suggest a Critical Point

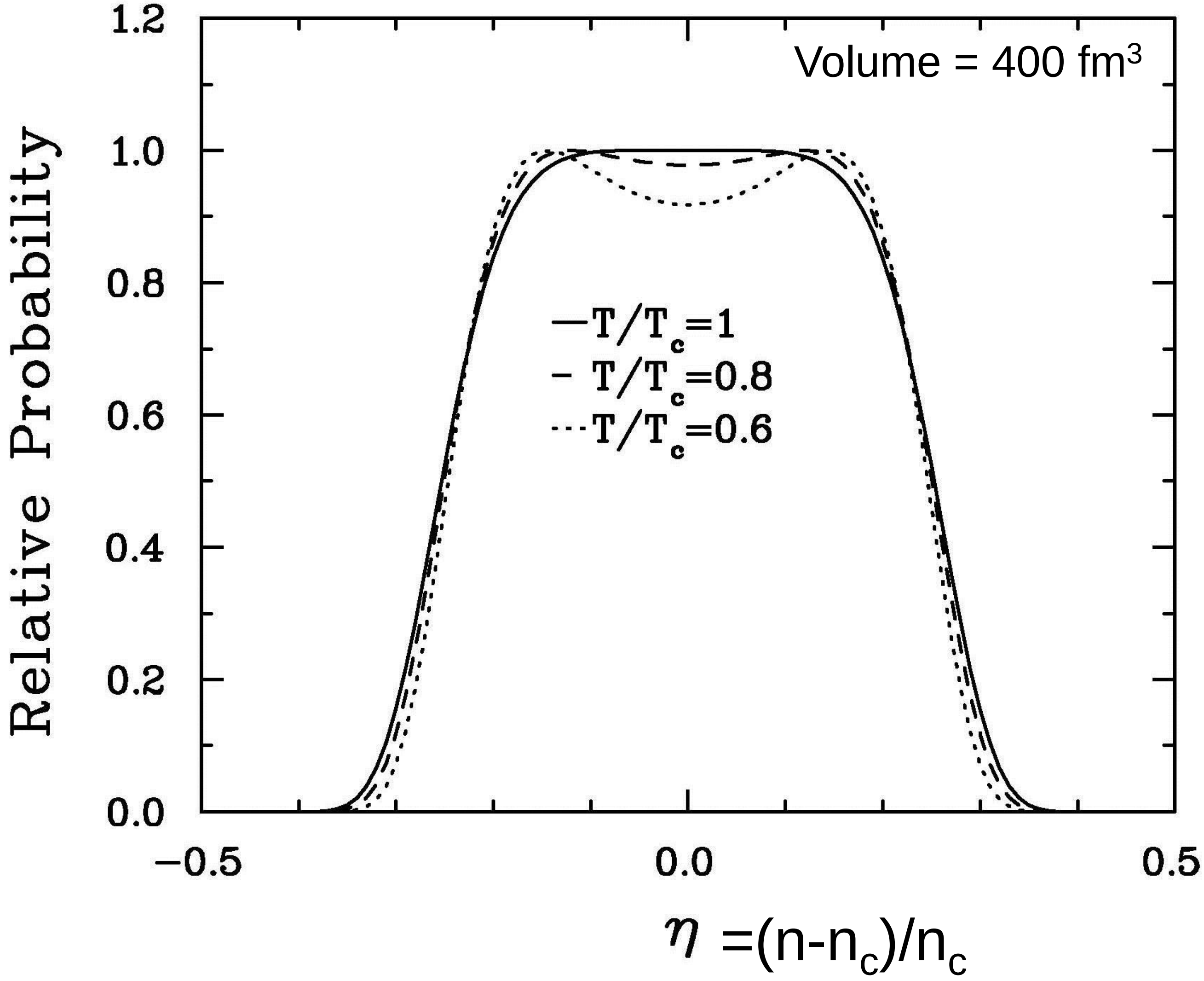
- Nambu Jona-Lasinio model
- composite operator model
- random matrix model
- linear sigma model
- effective potential model
- hadronic bootstrap model
- lattice QCD


The Phases of QCD
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
NSAC 2007 Long-range Plan
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
Crossover
~170 MeV
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Nuclear Matter
Neutron Stars
Vacuum
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

Many Accelerators

- Relativistic Heavy Ion Collider (RHIC) low energy runs - BNL
- Facility for Antiproton and Ion Research (FAIR) - Germany
- SPS Heavy Ion and Neutrino Experiment (SHINE) - CERN
- Nuclotron-based Ion Collider Facility (NICA) - Dubna

Expansion away from equilibrium states using Landau theory



Incorporates correct critical exponents and amplitudes - Kapusta (2010)
Static univerality class: 3D Ising model & liquid-gas transition

But this is for a small system
in contact with a heat and
particle reservoir.

How do you treat fluctuations
in an expanding and cooling
system as in heavy ion collisions?

Hydrodynamic Fluctuations!

Hydrodynamic fluctuations (noise) have been applied to a wide variety of physical, chemical, and biological systems.

There are fluctuations in high energy heavy ion collisions due to the finite size and finite particle content of the system.

VOLUME 89, NUMBER 8 PHYSICAL REVIEW LETTERS 19 AUGUST 2002

Dynamics of Liquid Nanojets

Jens Eggers

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(Received 31 January 2002; published 6 August 2002)

We study the breakup of a liquid jet a few nanometers in diameter, based on a stochastic differential equation derived recently by Moseler and Landman [Science **289**, 1165 (2000)]. In agreement with their simulations, we confirm that noise qualitatively changes the characteristics of breakup, leading to symmetric profiles. Using the path integral description, we find a self-similar profile that describes the most probable breakup mode. As illustrated by a simple physical argument, noise is the driving force behind pinching, speeding up the breakup to make surface tension irrelevant.

PRL **98,** 064504 (2007) PHYSICAL REVIEW LETTERS week ending 9 FEBRUARY 2007

Universality Crossover of the Pinch-Off Shape Profiles of Collapsing Liquid Nanobridges in Vacuum and Gaseous Environments

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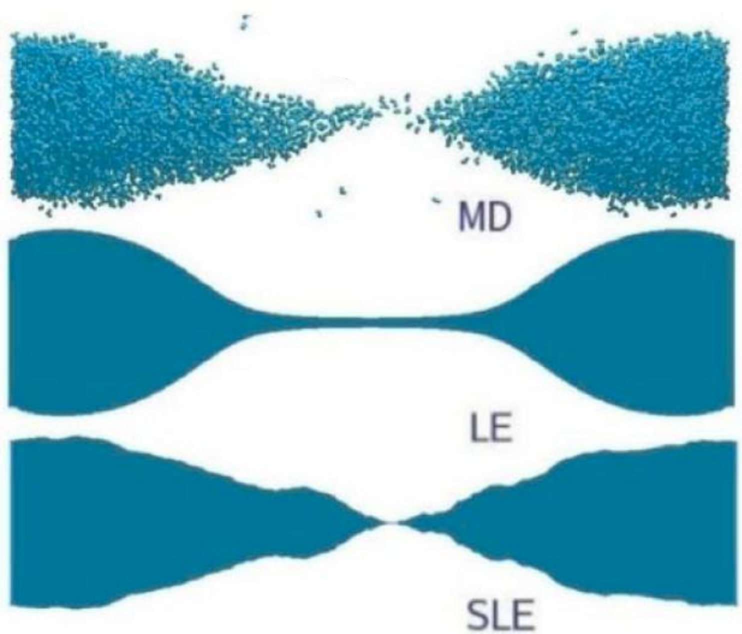
(Received 5 June 2006; published 7 February 2007)

Liquid propane nanobridges were found through molecular dynamics simulations to exhibit in vacuum a symmetric break-up profile shaped as two cones joined in their apexes. With a surrounding gas of sufficiently high pressure, a long-thread profile develops with an asymmetric shape. The emergence of a long-thread profile, discussed previously for macroscopic fluid structures, originates from the curvature-dependent evaporation-condensation processes of the nanobridge in a surrounding gas. A modified stochastic hydrodynamic description captures the crossover between these universal break-up regimes.



Molecular Dynamics

Lubrication Equation

Stochastic Lubrication Equation

Extend Landau's theory of hydrodynamic fluctuations to the relativistic regime

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \Delta T^{\mu\nu} + S^{\mu\nu}, \quad J^{\mu} = nu^{\mu} + \Delta J^{\mu} + I^{\mu}$$

Stochastic sources $S^{\mu\nu} = S^{\mu\nu}$ and I^{μ}

$$\left\langle S^{\mu\nu}(x) S^{\alpha\beta}(y) \right\rangle = 2T\left[\eta\left(h^{\mu\alpha}h^{\nu\beta} + h^{\mu\beta}h^{\nu\alpha}\right) + \left(\zeta - \tfrac{2}{3}\eta\right)h^{\mu\nu}h^{\alpha\beta}\right]\delta^4(x-y)$$

$$\left\langle S^{\mu\nu}(x) I^{\alpha}(y) \right\rangle = 0$$

$$\left\langle I^{\mu}(x) I^{\nu}(y) \right\rangle = 2\lambda\left(\frac{nT}{w}\right)^2 h^{\mu\nu}\delta^4(x-y)$$

Here we focus on thermal conductivity and set viscosities to zero.

The opposite case of n=0 and nonzero viscosities was studied by Kapusta, Mueller, Stephanov.

Mode coupling theory – diffusive heat and viscous are slow modes, sound waves are fast modes

Fixman (1962) Kawasaki (1970,1976) Kadanoff & Swift (1968) Zwanzig (1972) Luettmer-Strathmann, Sengers & Olchowy (1995) together with Kapusta (2010)

$$\Delta\lambda = c_p \Delta D_T = c_p \frac{R_D T}{6\pi\eta\xi}\Omega(q_D \xi)$$

= specific heat x Stokes-Einstein diffusion law x crossover function

$$\xi(n,T) \equiv \bar{\xi}_0 \left[\left(\frac{\delta - 1}{2-\gamma}\right)\left(\frac{\Delta n}{n_c}\right) t^{\gamma} + 5\delta\,|\eta|^{\delta-1} \right]^{-\nu/\gamma}$$

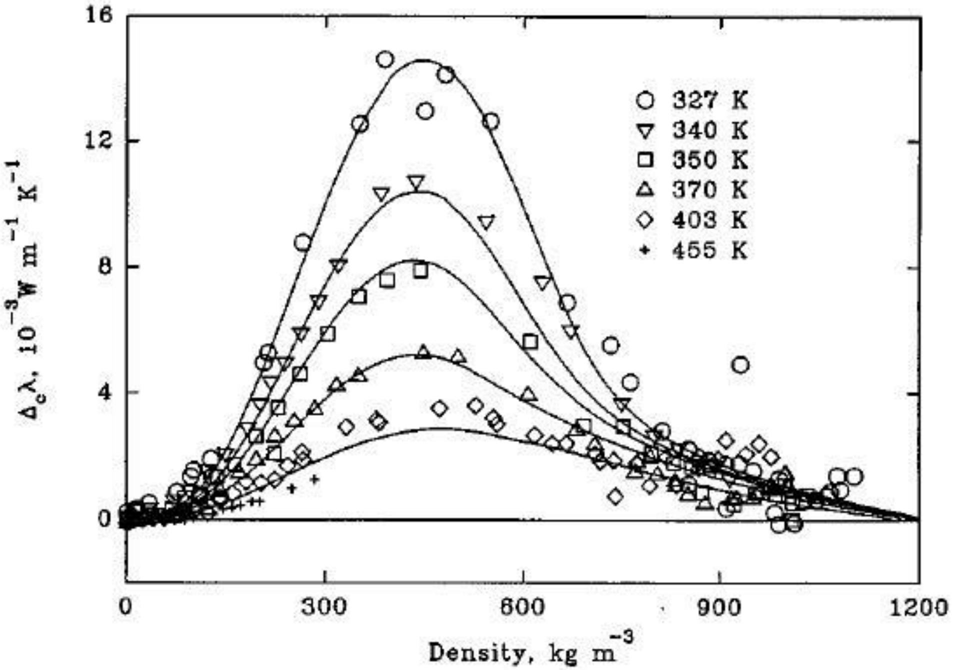
Critical exponent in reduced temperature t for $\Delta\lambda$ is $\gamma - \nu \approx 0.61$

Estimate $\bar{\xi}_0 \approx 0.69\ \mathrm{fm}$

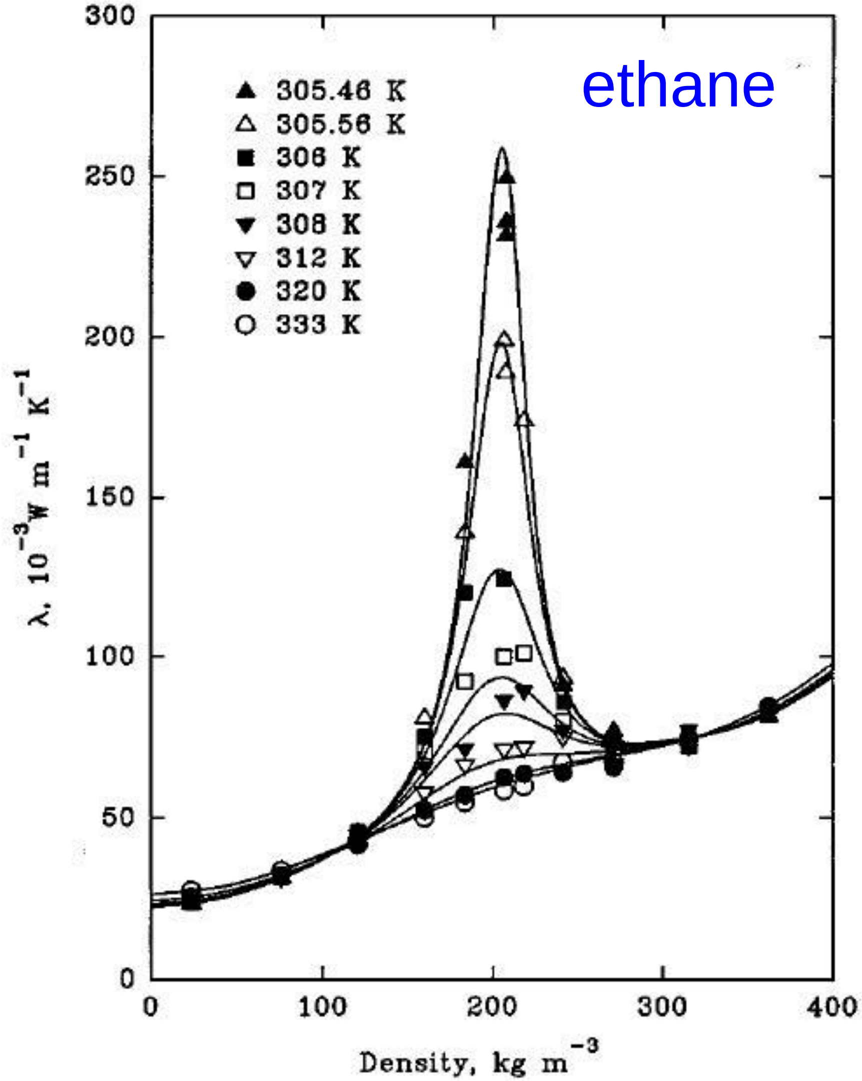
Dynamic universality class: Model H of Hohenberg and Halperin

Luettmer-Strathmann, Sengers & Olchowy (1995)

carbon dioxide

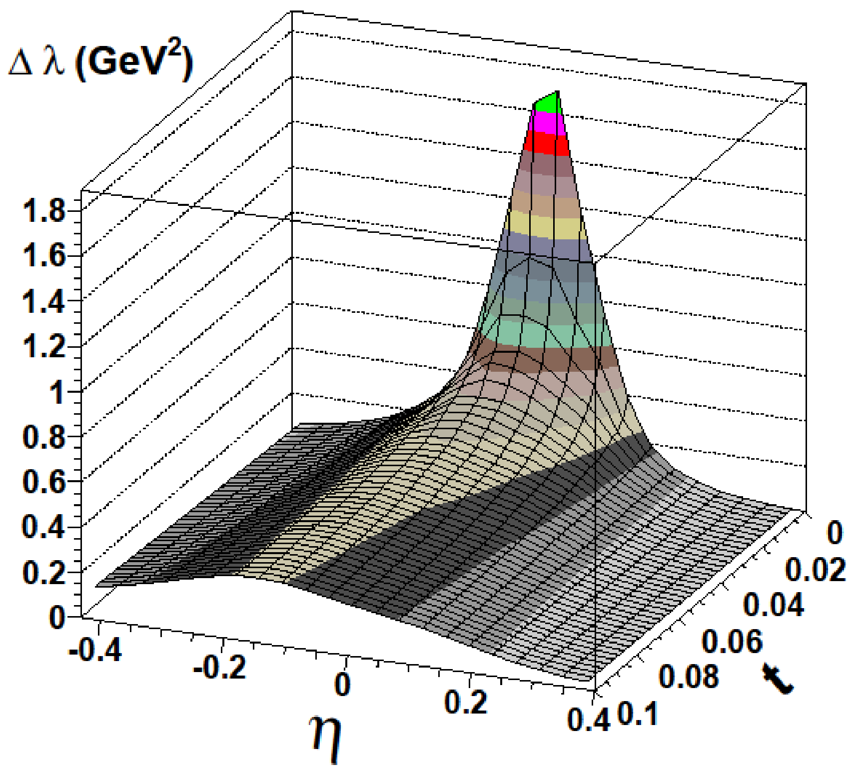


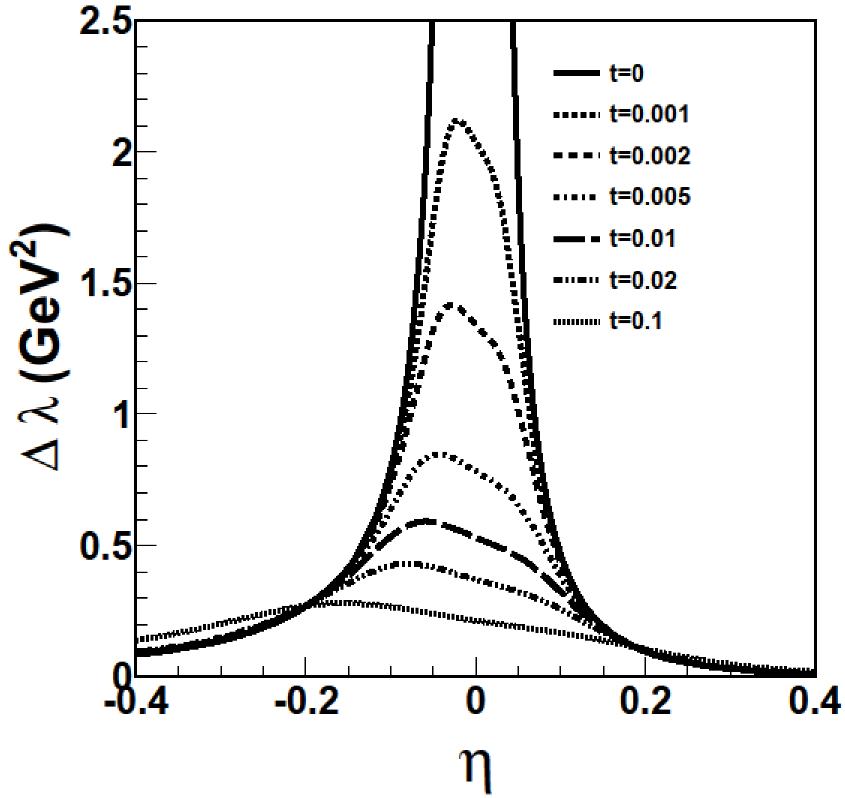
ethane



Data from various experimental groups.

Excess thermal conductivity





Will hydrodynamic fluctuations have an impact on our ability to discern a critical point in the phase diagram (if one exists)?

Procedure

- Solve equations of motion for arbitrary source function
- Perform averaging to obtain correlations/fluctuations
- Stochastic fluctuations need not be perturbative
- Need a background equation of state

$$P = A_4 T^4 + A_2 T^2 \mu^2 + A_0 \mu^4 - C T^2 - B$$

Simple Example: Boost Invariant Model

$$u^3 = \sinh(\xi + \omega(\xi,\tau)) \qquad n = \frac{n_i \tau_i}{\tau} + \delta\mathrm{n}(\xi,\tau) \qquad s(\tau) = \frac{s_i \tau_i}{\tau} + \delta\mathrm{s}(\xi,\tau)$$

Linearize equations of motion in fluctuations

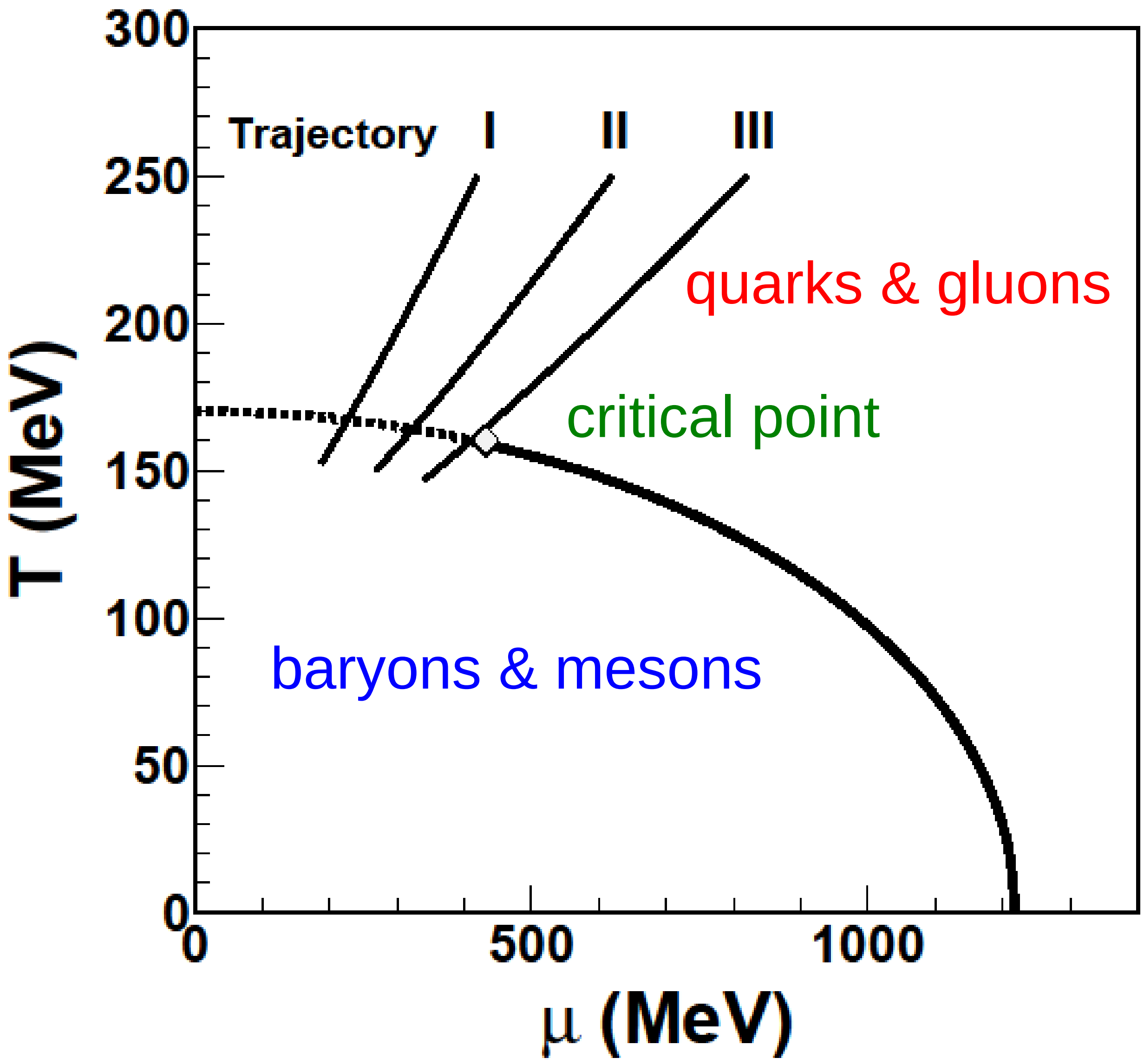
noise $\quad I^3 = s(\tau) f(\xi,\tau) \sinh\xi$

Solution: $$\tilde{X}(k,\tau) = -\int_{\tau_i}^{\tau} \frac{d\tau'}{\tau'} \tilde{G}_X(k;\tau,\tau') \tilde{f}(k,\tau') \qquad X = \delta n, \delta s, \omega$$

response function

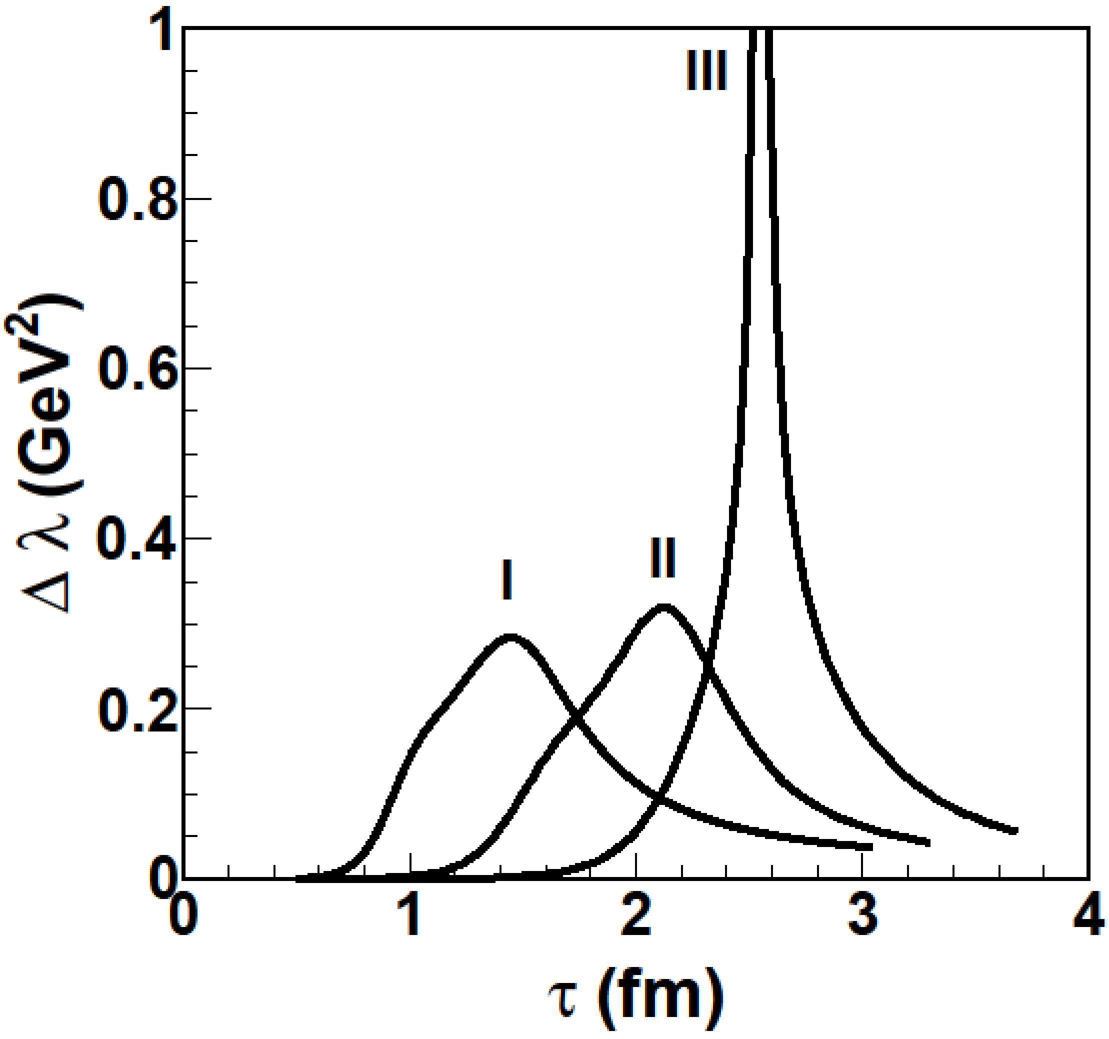
$$C_{XY}(\xi,\tau_f) = \frac{2}{A} \int_{\tau_i}^{\tau_f} \frac{d\tau}{\tau^3} \lambda(\tau) \left[\frac{n(\tau)T(\tau)}{s(\tau)w(\tau)} \right]^2 G_{XY}(\xi;\tau_f,\tau)$$

enhanced near critical point



300
250
200
150
100
50
0
0
500
1000
T (MeV)
μ (MeV)
Trajectory I II III
quarks & gluons
critical point
baryons & mesons

Excess thermal conductivity on the flyby



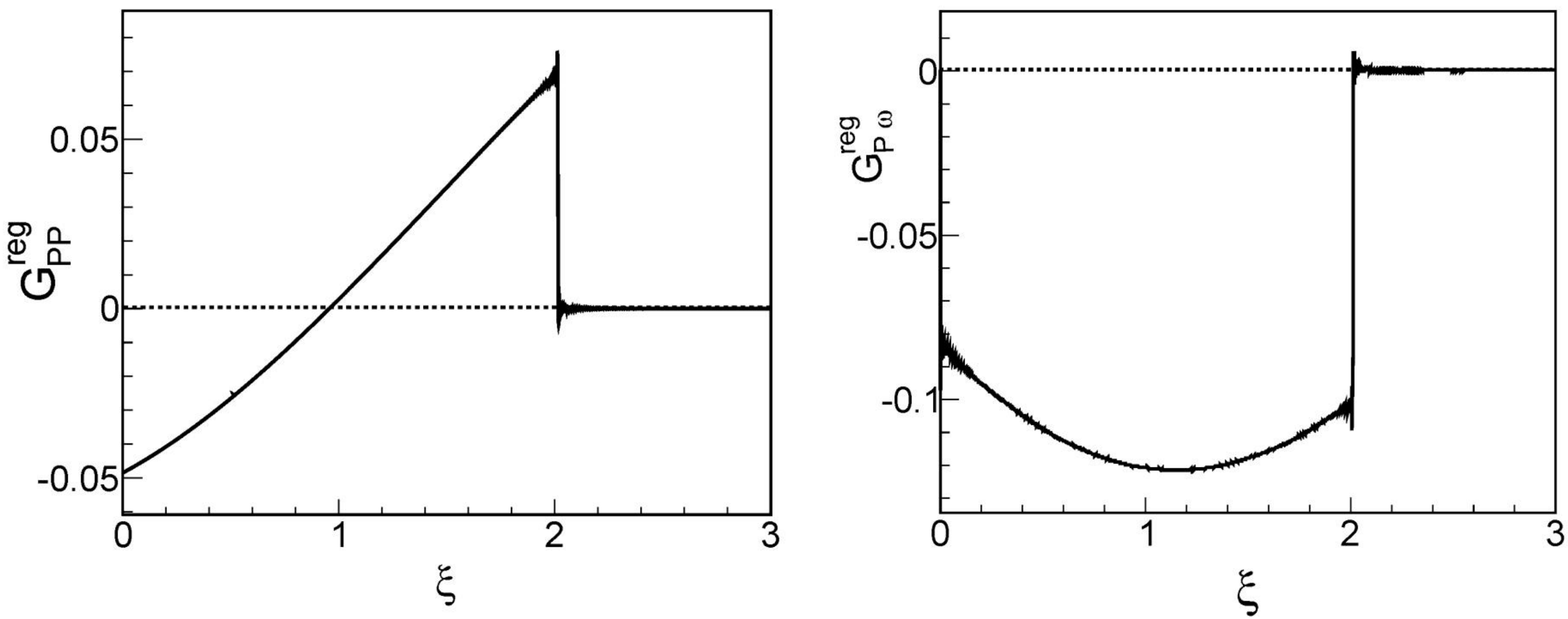
Fluctuations in the local temperature, chemical potential, and flow velocity fields

$$u_z = \sinh\left(\xi + \omega(\xi, \tau)\right)$$

give rise to a nontrivial 2-particle correlation function when the fluid elements freeze-out to free-streaming hadrons.

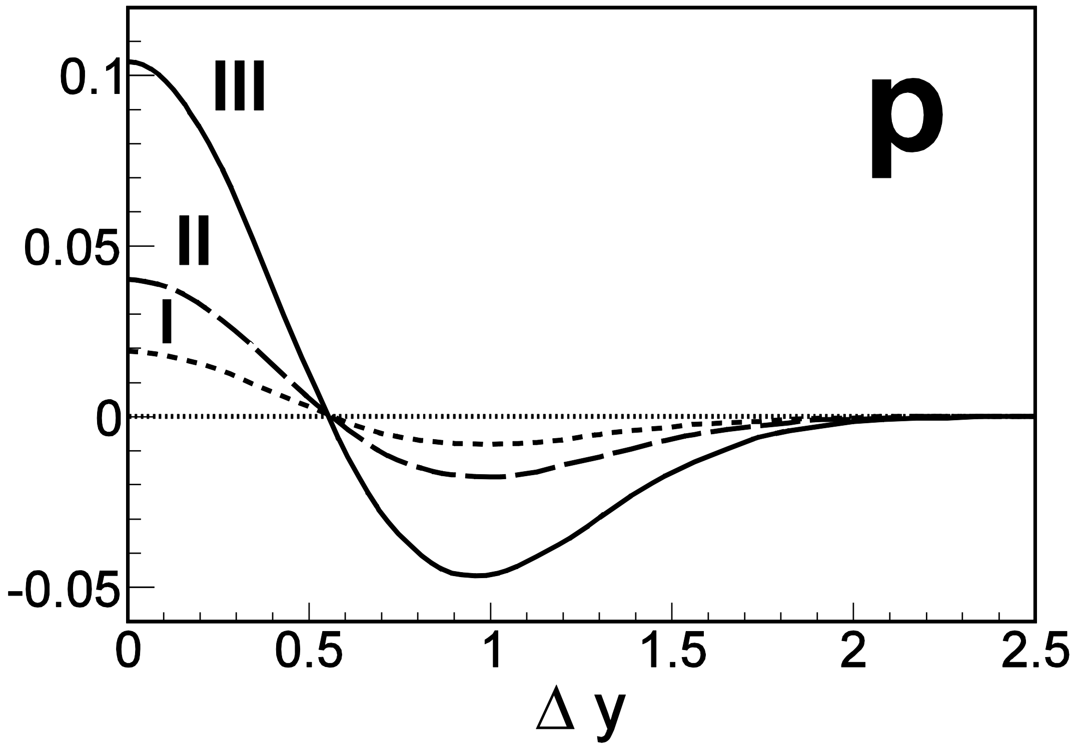
Regular part of the response function

Correlations are cut - off at the sound horizon $\xi = 2v_s \ln(\tau_f / \tau_i)$ with singular terms there and at $\xi = 0$ and a wake left behind.



Magnitude of proton correlation function depends strongly on how closely the trajectory passes by the critical point.

$$\left\langle \frac{dN(y_2)}{dy_2}\frac{dN(y_1)}{dy_1} - \left\langle \frac{dN}{dy} \right\rangle^2 \right\rangle \left\langle \frac{dN}{dy} \right\rangle^{-1}$$

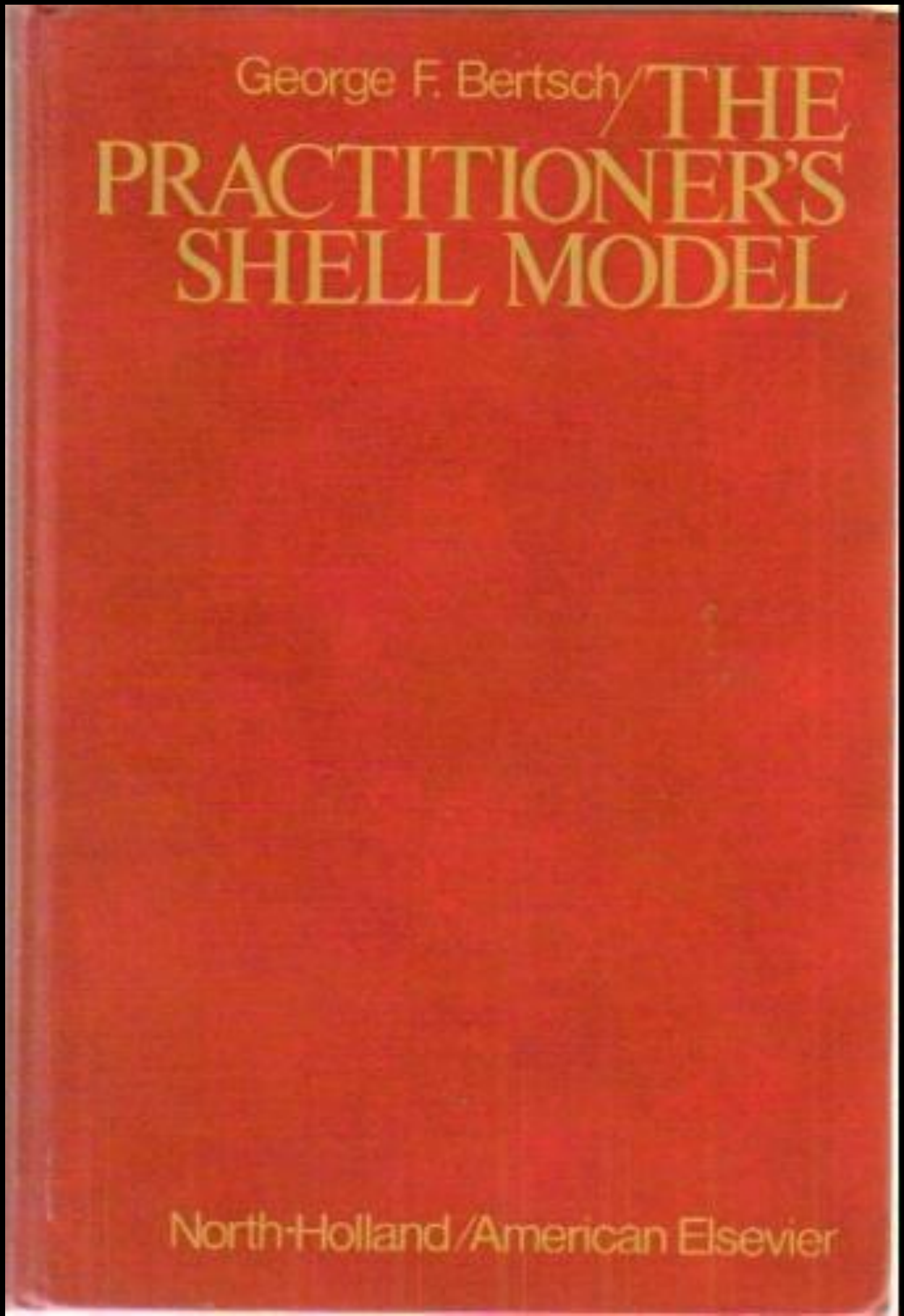


Summary

- Fluctuations are interesting and provide essential information on the critical point.
- Fluctuations are enhanced on a flyby of the critical point.
- There is clearly plenty of work for both theorists and experimentalists!

Supported by the Office Science, U.S. Department of Energy.

1972


George F. Bertsch / THE PRACTITIONER'S SHELL MODEL
North-Holland / American Elsevier

PRL 1979

Evidence for a Soft Nuclear-Matter Equation of State

Philip J. Siemens[a] and Joseph I. Kapusta
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 3 August 1979)

The entropy of the fireball formed in central collisions of heavy nuclei at center-of-mass kinetic energies of a few hundred MeV per nucleon is estimated from the ratio of deuterons to protons at large transverse momentum. The observed paucity of deuterons suggests that strong attractive forces are present in hot, dense nuclear matter, or that degrees of freedom beyond the nucleon and pion may already be realized at an excitation energy of 100 MeV per baryon.

PRC 1981

Entropy production in high energy collisions

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Santa Barbara, California 93106

J. Cugnon
Institut de Physique, Université de Lîege, Lîege, Belgium
(Received 29 June 1981)

The entropy production in high-energy collisions is computed in a Monte Carlo cascade model. For collisions of ^{40}Ca on ^{40}Ca at 800 MeV/nucleon beam energy, the computed entropy is 4.4 per particle, about a unit higher than estimated from bulk dynamics. The particle correlation function of the final state is also computed, and is found to be in reasonable accord with a thermal distribution of the same entropy. With such low entropy values, most of the particles emerge in clusters, contrary to experiment. Thus the cascade calculation supports the conclusion of Siemens and Kapusta, that additional degrees of freedom become accessible in heavy ion collisions, beyond those in a conventional nuclear description.

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